

AD-A113 997

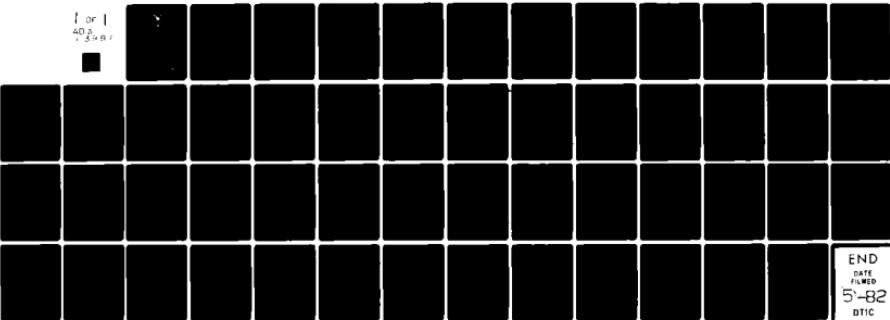
AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OH SCHOO--ETC F/6 12/1  
INCORPORATION OF ASYMPTOTIC NORMALITY PROPERTIES OF THE BINOMIA--ETC(U)  
DEC 79 R E RICE

UNCLASSIFIED

AFIT/GOR/MA/79D-8

NL

For I  
402  
1-8-82

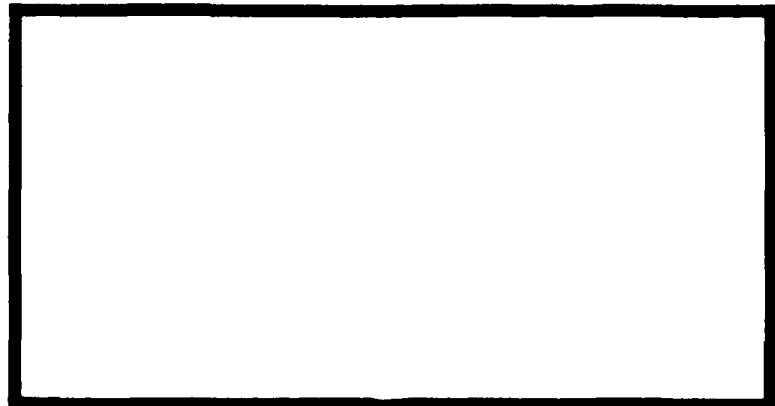


END  
DATE  
FILED  
5-82  
DTIC

(1)



AD A113997



UNITED STATES AIR FORCE  
AIR UNIVERSITY  
**AIR FORCE INSTITUTE OF TECHNOLOGY**  
Wright-Patterson Air Force Base, Ohio

DTIC  
ELECTED  
APR 29 1982  
S D

82 04 29 068 E

DTIC FILE COPY

(1)

AFIT/GOR/MA/79D-8

INCORPORATION OF ASYMPTOTIC NORMALITY  
PROPERTIES OF THE BINOMIAL DISTRIBUTION  
INTO A MONTE CARLO TECHNIQUE FOR  
ESTIMATING LOWER CONFIDENCE LIMITS  
ON SYSTEM RELIABILITY

THESIS

AFIT/GOR/MA/79D-8

Roy E. Rice  
Capt USAF

Approved for public release; distribution unlimited

AFIT/GOR/MA/79D-8

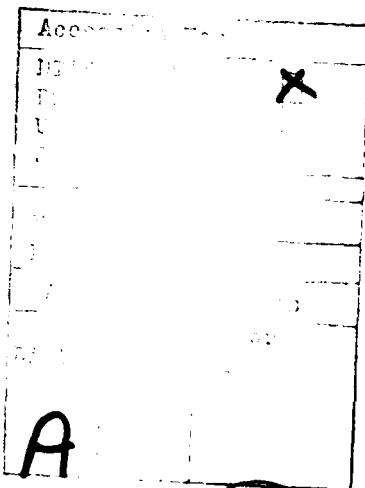
INCORPORATION OF ASYMPTOTIC NORMALITY PROPERTIES  
OF THE BINOMIAL DISTRIBUTION INTO A MONTE CARLO  
TECHNIQUE FOR ESTIMATING LOWER CONFIDENCE  
LIMITS ON SYSTEM RELIABILITY

THESIS

Presented to the Faculty of the School of Engineering  
of the Air Force Institute of Technology  
Air University  
in Partial Fulfillment of the  
Requirements for the Degree of  
Master of Science

by  
Roy E. Rice, B.S.  
Capt USAF  
Graduate Operations Research

December 1979



Approved for public release; distribution unlimited.

Preface

Many methods of obtaining confidence limits for system reliability exist in both practice and literature. The understanding and use of these techniques are often quite difficult. Because of this, I embarked on an effort to devise a technique that was theoretically sound, easy to understand, and simple to use. This report discusses that technique and its accuracy.

Sincere gratitude is extended to Dr. Albert H. Moore for his guidance, expertise, and motivation toward completion of this study. Thanks also goes to Mr. Joseph J. Meli, ASD/ENADC, who was my sponsor.

Roy E. Rice

## Contents

	<b>Page</b>
Preface . . . . .	ii
List of Figures . . . . .	iv
List of Tables . . . . .	v
Abstract . . . . .	vi
I. Introduction . . . . .	1
Background . . . . .	1
Approach . . . . .	1
Assumptions . . . . .	3
II. Background of Estimating Techniques . . . . .	4
Mellin Transforms . . . . .	4
Approximate Optimality . . . . .	5
Gatliffe's Method . . . . .	7
III. Proposed Method . . . . .	11
Parameter Estimates . . . . .	11
Monte Carlo Simulation . . . . .	12
Outline of Procedure . . . . .	13
Results . . . . .	14
IV. Verification of Accuracy . . . . .	19
Parameter Estimates . . . . .	19
Monte Carlo Simulation . . . . .	20
Outline of Procedure . . . . .	21
Results . . . . .	22
Examples . . . . .	25
V. Conclusions and Recommendations . . . . .	29
Bibliography . . . . .	30
Appendix A: Case of $\hat{s}=0$ . . . . .	33
Appendix B: Computer Program to Determine Limits . .	36
Appendix C: Computer Program to Verify Accuracy . .	39
Vita . . . . .	44

List of Figures

<u>Figure</u>		<u>Page</u>
1	Three Component System . . . . .	27
2	Five Component System . . . . .	27

List of Tables

<u>Table</u>		<u>Page</u>
I	90% Limits, Two Components . . . . .	15
II	95% Limits, Two Components . . . . .	16
III	90% Limits, Three Components . . . . .	17
IV	95% Limits, Three Components . . . . .	18
V	Two Component System . . . . .	23
VI	Three Component System . . . . .	24

Abstract

Several methods for estimating lower confidence limits were examined. A proposed technique was developed and used to obtain limits for selected systems. These limits were compared to those obtained by other methods. Then the proposed method was tested to verify its accuracy. Results indicate that the proposed method is simple to understand, easy to implement, and accurate.

INCORPORATION OF ASYMPTOTIC NORMALITY PROPERTIES OF  
THE BINOMIAL DISTRIBUTION INTO A MONTE CARLO  
TECHNIQUE FOR ESTIMATING LOWER CONFIDENCE LIMITS  
ON SYSTEM RELIABILITY

I. Introduction

In the late 1920's and early 1930's, prominent statisticians began to realize that a single point estimate for the probability of occurrence of an event was not sufficient. Initial studies by Fischer, Neyman, Clopper, Pearson, and many others show the necessity and practicality of placing probabilistic bounds on these estimates. The application of these intervals injects confidence into the accuracy of these point estimates. Through these pioneering efforts we arrived at the concept of confidence intervals.

This analysis intends to introduce some of the more important historical works that address the calculation of confidence limits. It contains considerably detailed discussions of the evolution of various techniques for deriving confidence limits with major emphasis on intervals for binomial distributed parameters. There are discussions of techniques involving Bayesian Limits using Mellin Transforms (Ref 22:611), Approximately Optimum Limits (Ref 18: 335-347), Monte Carlo simulation (Ref 19:459), and others. There is then a discussion of the proposed method of incorporating the asymptotic normality assumption into a Monte

Carlo technique. Finally, a verification technique is included that shows how accurate the confidence limits are.

Component test data is obtained where the number of failures is assumed to be distributed binomially. Then the components are arranged in a logically complex system and an estimate of the reliability and confidence limit is made. These estimates from the proposed method are then compared with estimates from the other methods. A final effort is made to verify the accuracy of the limits by simulating the testing of the components and determining the number of times the limits cover the true reliability.

Of particular interest is the treatment of test data in which no failures occur. Many of the methods are not valid when the data shows zero failures. Others simply disregard the samples with zero failures. Still others manipulate the data to circumvent the problem. Almost all of this results in a biasing effect on the estimates. This study explores another way of handling samples that exhibit no failures.

Coupled with the improvement of these estimating techniques is the advancing computer technology. High speed computers make it increasingly easier and more efficient to make detailed computations and simulations. Thus, it is becoming more practical to apply these methods to systems exhibiting pass/fail characteristics from the simple (switches, pins, relays) to the very complex (missile flights, docking systems, targeting).

In the following study there are several basic assumptions. It is assumed that the reader is familiar with the concept of confidence limits, the binomial distribution, and has a basic understanding of statistics and reliability pertaining to both components and systems.

## II. Background of Estimating Techniques

In one of the first studies in this area, Springer and Thompson (Ref 22:611) examined the posterior density functions of  $R_i$  ( $i=1,2,\dots,N$ ), the binomial parameters of  $N$  independent subsystems, when the prior distribution is uniform or Beta. The Bayes posterior density function of  $R_i$  in the uniform prior case is

$$f_i(R_i) = \frac{(n_i+1)!}{m_i!(n_i-m_i)!} R_i^{m_i} (1-R_i)^{n_i-m_i}$$
$$(0 \leq R_i \leq 1, i=1,2,\dots,N)$$

where  $m_i$  = number of successes in  $n_i$  trials.

Epstein gives the Mellin integral transform of  $f_i(R_i)$  as

$$M\{f_i(R_i)|s\} = \frac{(n_i+1)!}{m_i!} \frac{(s+m_i)}{(s+n_i+1)}, R_e(s) > m_i.$$

Huntington shows that the Mellin convolution of  $f_1(R_1)$  and  $f_2(R_2)$  is exactly the probability density function of the product  $R_1 R_2$  and that the repeated convolution of the probability density functions is equivalent in the transform to repeated multiplication of the Mellin transforms. From this, Springer and Thompson derive a closed form of the posterior distribution function for  $N$  independent subsystems in series,  $H(R)$ . At this point, they emphasize that, for large  $N$ , computation of  $H(R)$  is tedious for hand

computations and extensive using computers. Thus, they conclude that some asymptotic method might warrant investigation. For more detailed investigation of their method, see Reference 22.

Later, in a study by Mann and Grubbs (Ref 18:335-347), confidence bounds for system reliability are derived based on component test data when the components are assumed to be (1) exponentially distributed with censoring or truncation of tests for a fixed number of failures, (2) exponentially distributed with truncation of tests at fixed times, and (3) binomially distributed (pass-fail) with fixed but different sample sizes and random numbers of failures for subsystem tests. With regard to the binomial distribution, Mann arrives at approximate expressions for  $m_s$  and  $v_s$ , the mean and variance of  $-\ln R_s$  which corresponds to these optimum nonrandomized confidence bounds for series systems.

These are

$$m_s = \frac{.5(1+\frac{1}{a})}{n^o} + \frac{\hat{p}}{(1-.5\hat{p})}$$

and

$$v_s = \frac{.5(1-\frac{1}{a})m_s}{n^o},$$

where  $n^o = n_{(1)}(1-.5\hat{p}^2)(1-.5\hat{p})$

$n_{(1)}$  is the smallest of the  $n_1, n_2, \dots, n_k$ .

$\hat{P} = 1 - \prod_{j=1}^k \hat{R}_j$  is the max-likelihood estimate of the probability of failure of the series system.

$a = n_{(1)} \sum (1/n_j)$  which is a restricted sum.

The  $n_j$ 's for all zero-failure subsystems are omitted in calculating the value of  $a = n_{(1)} \sum (1/n_j)$  except for a single zero-failure subsystem with its sample size equal to  $n_{(1)}$ . Mann and Grubbs point out that this subsystem is also ignored if at least one of the other subsystems which exhibit some failures has the smallest sample size  $n_{(1)}$ , or if the lower confidence bound obtained by not ignoring it is larger than that obtained by ignoring it.

"The exclusion of some subsystems in calculating series-system confidence bounds also is necessitated when all sample sizes are large and most of the failures pertain to a single subsystem. In this case the bound obtained based on all subsystem failures can be larger than that based on the single subsystem exhibiting nearly all the failures." (Ref 18:337)

In their analysis they compare their bounds with those based on Buehler's exact methods, those based on maximum likelihood normal approximations, the modified maximum likelihood procedure of Easterling, and the likelihood ratio method of Madansky. These last three procedures cannot be applied effectively when one or more subsystems exhibit zero-failures. This comparison, along with the estimates from the proposed method, is shown in Tables I, II, III, and IV.

Another technique that is of extreme importance is that developed by Gatliffe (Ref 7). Mathematically, Gatliffe begins by stating that, for systems with  $k$  components in series,

$$R_s = \prod_{i=1}^k P_i , P_i \text{ is the reliability of the } i^{\text{th}} \text{ component}$$

$$Q_i = 1 - P_i .$$

$$\text{Let } s = -\ln R_s = -\sum_{i=1}^k \ln(1 - Q_i)$$

$$s = \sum_{i=1}^k \left( Q_i + \frac{Q_i^2}{2} + \frac{Q_i^3}{3} + \dots \right)$$

$$\approx \sum_{i=1}^k \left( Q_i + \frac{Q_i^2}{2} \right) = \sum_{i=1}^k T_i , T_i = \left( Q_i + \frac{Q_i^2}{2} \right) .$$

The unbiased estimator,  $\hat{T}_i = A_i \hat{Q}_i + B_i (\hat{Q}_i^2/2)$

$$A_i = \frac{(2N_i - 3)}{(2N_i - 2)}$$

$$B_i = \frac{N_i}{N_i - 1} ,$$

$$Q_i = \frac{F_i}{N_i} , F_i = \text{the number of failures resulting from } N_i \text{ mission tests conducted on } i^{\text{th}} \text{ component.}$$

He establishes another unbiased estimator,

$$\hat{s} = \sum_{i=1}^k \hat{T}_i$$

with  $\text{Var}(\hat{s}) = \sum_{i=1}^k (T_i/N_i)$ .

He intuitively assumes the probability distribution of  $\hat{s}$  is gamma with parameters  $r$  and  $\theta$ ,

$$f_{\hat{s}}(x, r, \theta) = \begin{cases} \frac{1}{\Gamma_r \cdot \theta^r} x^r e^{-x/\theta}, & x \geq 0 \\ 0 & , x < 0 \end{cases}$$

with mean =  $r\theta$ , variance =  $r\theta^2$ .

$$E(\hat{s}) = r\theta = \sum_{i=1}^k T_i$$

$$\text{Var}(\hat{s}) = r\theta^2 = \sum_{i=1}^k (T_i/N_i)$$

$$r = \frac{(\sum T_i)^2}{\sum \frac{T_i}{N_i}}$$

$$\theta = \frac{\sum T_i / N_i}{\sum T_i}$$

$$\hat{r} = \frac{(\sum \hat{T}_i)^2}{\sum \hat{T}_i / N_i}.$$

Since  $\hat{s}$  is distributed as gamma ( $r, \hat{\theta}$ ),  $\frac{2\hat{s}}{\theta}$  will be  $\chi^2_{(2r)}$ .

From

$$1 - \alpha = P\left[\frac{2\hat{s}}{\theta} \geq \chi^2_{(2r, 1-\alpha)}\right] \text{ and}$$

$$1 - \alpha = P\left[\theta \leq \frac{2\hat{s}}{\chi^2_{(2r, 1-\alpha)}}\right] \text{ and since } \theta = \frac{E(\hat{s})}{r} \text{ and}$$

$E(\hat{s}) = -\ln(R_s)$ , he has obtained a  $100(1-\alpha)\%$  lower confidence limit,  $R_s^*(\alpha)$ , for  $R_s$

$$R_s^*(\alpha) = \exp\left[\frac{-2rs}{\chi^2_{(2r, 1-\alpha)}}\right].$$

A  $100(1-\alpha)\%$  lower confidence limit estimate,  $\hat{R}_s^*(\alpha)$ , for  $R_s$  would be

$$\hat{R}_s^*(\alpha) = \exp\left[\frac{-2\hat{r}\hat{s}}{\chi^2_{(2r, 1-\alpha)}}\right].$$

It is apparent that a problem arises in the event of no failures in any components. This yields an  $\hat{s}$  value of zero, thus, a system reliability confidence limit estimate of one (1).

Gatliffe outlines his procedure to deal with such an event as follows:

(1) Find the series component with the smallest actual or equivalent number of trials,  $N$  or  $N'$ .

(2) Change the number of component or subassembly failures to  $F'$  by the following scheme:

<u><math>\alpha</math></u>	<u><math>F'</math></u>
.2	.37
.1	.25
.15	.16

(See derivation in Appendix A.)

(3) Recompute the lower confidence limit estimate using the basic log-gamma procedure with the revised failure data. This estimate is approximately equivalent to  $(\alpha)^{1/N}$ .

Gatliffe's scheme to arrive at equivalent failures was used in the proposed method. It is an amazingly simple, yet effective tool; and, for this reason, it is recommended that the reader refer to Appendix A.

The techniques discussed in this chapter were not used directly in the proposed method. Yet, they give different approaches that are of historical and analytical importance. They are primarily doorways that open up into a vast and intriguing area of study.

### III. Proposed Method

The essence of this study is to examine a series-system with components that experience failures distributed binomially and derive a simple, yet accurate, method for estimating confidence limits for system reliability. The method of obtaining lower confidence limits that is proposed in this study draws upon two important concepts; the asymptotic normality properties of the binomial distribution and Monte Carlo simulation. Both warrant discussion.

It has been shown that, when a pass-fail component is tested a sufficient number of times (20 or more), its failures are distributed normally with a mean of  $\hat{p}$  and a variance of  $\frac{\hat{p}\hat{q}}{n}$ . In this treatment,  $\hat{p}$  is defined as the probability of success,  $\hat{q}$  is  $1-\hat{p}$ , and  $n$  is the number of trials. So for any component  $i$ ,

$$\hat{p}_i = 1 - \frac{F_i}{n_i}, F_i = \text{the number of failures of } i^{\text{th}} \text{ component in } n_i \text{ trials,}$$

$$\hat{q}_i = 1 - \hat{p}_i.$$

Note that if there are no failures,  $\hat{p}_i = 1$  and the variance equals zero. The existence of components with a reliability absolutely equal to 1 is, to say the least, a rarity. To eliminate this problem, an equivalent number of failures,  $F'_i$ , is assigned according to Gatliffe's method (see

Appendix A). Once these initial estimates have been obtained, they can be used in a Monte Carlo simulation.

This Monte Carlo method begins by assigning failures or equivalent failures to each component in a series system. From this a  $\hat{p}_i$  is obtained for each component. Now, using the asymptotic normality property, a random variable from a normal distribution, with mean = 0 and variance = 1, is drawn. If  $\frac{\hat{p}_i \hat{q}_i}{n_i}$  is the asymptotic variance, then the square root of this value is the standard deviation. Since a  $N(0,1)$  random variable is

$$z = \frac{\hat{p}_i - \mu}{\delta},$$

one can take this random variable, multiply it by the asymptotic standard deviation, and add it to the mean,  $\hat{p}_i$ . What is obtained is a second estimate of  $p_i$ ,

$$\hat{\hat{p}}_i = N(\hat{p}_i, \frac{\hat{p}_i \hat{q}_i}{n_i}).$$

If this is done for each component, one can obtain an estimate of the system reliability for k components in series,

$$\hat{\hat{R}}_s = \prod_{i=1}^k \hat{\hat{p}}_i.$$

A Monte Carlo simulation (Ref 9,10,15,19,20,21) is used to generate a number of these system reliability estimates. These estimates are then ranked in ascending order.

The ordered reliabilities partition the interval [0,1] into  $n+1$  equally probable intervals. For simplicity, let  $n+1$  equal 1000. From the ordered simulation reliabilities one can obtain any desired lower confidence limit by extracting the  $100(1-\alpha)$  percentile value. For instance, with 999 ordered reliability estimates, the 90 percent lower confidence limit would be the 99th ordered estimate.

The outline of this procedure is as follows:

- (1) For each component in turn assign the number of failures,  $F_i$ , or equivalent failures,  $F'_i$ ,
- (2) Calculate the first estimate

$$\hat{p}_i = 1 - \frac{F_i}{n_i} \text{ or } 1 - \frac{F'_i}{n_i},$$

$$\hat{q}_i = 1 - \hat{p}_i,$$

$$\text{asymptotic variance} = \frac{\hat{p}_i \hat{q}_i}{n_i},$$

- (3) Draw a random variable from  $N(0,1)$  for each component,

- (4) Obtain a second estimate,  $\hat{\hat{p}}_i \sim N(\hat{p}_i, \frac{\hat{p}_i \hat{q}_i}{n_i})$  by taking the  $N(0,1)$  random variable, multiply by the asymptotic standard deviation, and add it to  $\hat{p}_i$  for each component,

- (5) With the components in a logically complex system, arrive at a system reliability estimate,  $\hat{R}_s = \prod_{i=1}^k \hat{\hat{p}}_i$ ,

- (6) Repeat steps (3) through (5) 999 times,
- (7) Rank order these 999 estimates of  $R_s$ ,
- (8) Extract the  $100(1-\alpha)$  percentile value to obtain a  $100(1-\alpha)\%$  lower confidence limit.

This procedure was applied to systems of 2 and 3 components in series. The computer program used (see Appendix B) shows that, with minor adjustments, larger systems can be examined. In these 2 and 3 component systems, various numbers of failures and tests were analyzed. These particular values were chosen so as to give a basis of comparison (see Tables I, II, III, and IV). The proposed method produces values that compare extremely well with those used in the Mann and Grubbs study (Ref 18:345).

Several points in the tables deserve note. First, the values obtained in this method most closely compare with those from the maximum likelihood method. Secondly, in every case, they are slightly higher than the "exact" values which tends to indicate upward biasness in the estimates. This latter aspect is what led to further investigation as to the accuracy of the estimates. The next chapter examines this area.

TABLE I  
90% Limits, Two Components

No. of Components=2	No. of Failures		90% LCL					
	f <sub>1</sub>	f <sub>2</sub>	AN/MC	ML	LR	OPT	AO	MMLI
n=10	1	1	.655	.629	.607	.606	.585	
	1	2	.542	.545	.529	.497	.493	.489
	2	2	.458	.456	.451	.445	.430	.441
	1	4	.337	.347	.350	.344	.335	.318
	2	3	.372	.373	.375	.354	.353	.362
	1	2	.754	.756	.739	.716	.728	.709
n=20	2	2	.700	.701	.687	.683	.678	.669
	1	3	.693	.697	.683	.660	.675	.655
	2	3	.646	.647	.636	.622	.628	.619
	3	3	.599	.599	.591	.585	.582	.570

AN/MC = Proposed Method (Asymptotic Normality/Monte Carlo)

ML = Maximum Likelihood

LR = Likelihood Ratio (Madansky)

OPT = Exact Lower Bound

AO = Approximation Optimum (Mann)

MMLI = Modified Max Likelihood (Easterling)

TABLE II  
95% Limits, Two Components

No. of Components=2	No. of Failures				95% LCL			
	f <sub>1</sub>	f <sub>2</sub>	AN/MC	ML	LR	OPT	AO	MMLI
n=10	1	1	.614	.611	.571	.548	.552	.530
	1	2	.495	.495	.473	.443	.435	.436
	2	2	.414	.405	.397	.392	.382	.391
	1	4	.290	.292	.301	.298	.293	.271
	2	3	.328	.320	.326	.304	.307	.315
	1	2	.724	.728	.700	.677	.693	.671
n=20	2	2	.669	.670	.647	.643	.643	.631
	1	3	.663	.665	.643	.620	.639	.616
	2	3	.612	.614	.597	.582	.593	.580
	3	3	.565	.565	.551	.544	.548	.532

TABLE III  
90% Limits, Three Components

No. of Components=3	No. of Failures			90% LCL					
	$f_1$	$f_2$	$f_3$	AN/MC	ML	LR	OPT	AO	MMLI
$n=20$	1	1	1	.757	.760	.743	.747	.741	.721
	1	1	2	.704	.704	.690	.693	.689	.669
	1	2	2	.653	.654	.643	.639	.643	.619
	1	2	3	.608	.605	.596	.595	.597	.587
	1	2	3	.724	.723	.714	.705	.716	.669
	1	1	1	.833	.835	.822	.825	.820	.803
$n=30$	2	2	2	.722	.725	.715	.712	.717	.703
	1	2	4	.806	.805	.798	.789	.800	.788
	1	1	2	.873	.874	.865	.861	.870	.852
	1	1	2	.935	.936	.931	.929	.932	.923
$n=100$	2	3	5	.866	.866	.861	.858	.865	.856

TABLE IV  
95% Limits, Three Components

No. of Components=3				95% LCL					
	f <sub>1</sub>	f <sub>2</sub>	f <sub>3</sub>	AN/MC	ML	LR	OPT	AO	MMLI
n=20	1	1	1	.725	.732	.705	.709	.708	.684
	1	1	2	.667	.673	.651	.644	.657	.631
	1	2	2	.613	.621	.604	.598	.609	.580
	1	2	3	.567	.571	.557	.544	.554	.549
	1	2	3	.693	.698	.683	.674	.691	.638
	1	1	1	.810	.816	.794	.796	.698	.775
n=30	2	2	2	.694	.700	.685	.681	.692	.672
	1	2	4	.784	.788	.776	.767	.781	.766
	1	1	2	.856	.860	.845	.841	.854	.833
	1	1	2	.926	.929	.920	.918	.923	.913
n=100	2	3	5	.852	.855	.848	.844	.854	.842

#### IV. Verification of Accuracy

To merely derive an estimating technique and its subsequent estimates is not sufficient. Analysis must be conducted to verify the accuracy of the method. As discussed in Chapter I of this study, the concept of confidence intervals and limits evolved from the need to know the accuracy of point estimates. But it is obvious upon examining Tables I, II, III, and IV that, depending on which method is used, values of these confidence limits vary considerably. This leads to an effort to verify the accuracy of the proposed method of obtaining lower confidence limits.

Basic to the theory of confidence limits is the fact that, if a system of known reliability is tested a number of times, the estimates of a  $100(1-\alpha)\%$  confidence interval should include the true system reliability  $100(1-\alpha)\%$  of the time. From this fact came the motivation for the subsequent analysis.

Given a system with known reliability that is made up of components with established probabilities of success, one can apply the procedures described in Chapter III to estimate desired lower confidence limits and compare these limits with the true system reliabilities. Then, by generating many of these limits, it can be determined how often the true reliability is in the interval  $[LCL, 1]$ . For example, if 1000 80% lower confidence limits are generated, it is expected, theoretically, that 800 of the limits are lower than the true reliability.

Monte Carlo simulation, again, is the tool. Given  $k$  components with known probabilities of success,  $p_i$ , connected in series, a true system reliability is calculated,

$$R_s = \prod_{i=1}^k p_i .$$

Next, one must specify the number of tests that each component is to be subjected to,  $NT_i$ . To simulate the testing of each component, simply draw a uniform random variable in the interval  $[0,1]$  for each test. Now compare each of these  $NT_i$  random variables with  $p_i$ . If the random variable is greater than  $p_i$ , a failure is recorded. By doing this for each component, one obtains the first estimate of  $p_i$ ,

$$\hat{p}_i = 1 - \frac{F_i}{NT_i} ,$$

$$\hat{q}_i = 1 - \hat{p}_i ,$$

$$\text{asymptotic variance} = \frac{\hat{p}_i \hat{q}_i}{NT_i} .$$

As done in the previous chapter, if no failures are experienced, Gatliffe's method is used to assign equivalent failures. This is done for each component. The proposed method then produces a second estimate of  $p_i$  by drawing a random variable from a  $N(0,1)$ , multiplying by the asymptotic standard deviation, and adding this to  $\hat{p}_i$ . These estimates,  $\hat{\hat{p}}_i$ , are then combined to produce an estimate of  $R_s$ ,

$$\hat{R}_s = \frac{k}{\pi} \sum_{i=1}^k \hat{p}_i .$$

Again, 999 of these estimates,  $\hat{R}_s$ , are generated and ranked in ascending order. Then the  $100(1-\alpha)$  percentile value is extracted to yield a  $100(1-\alpha)\%$  lower confidence limit estimate,  $\hat{R}_{s_j}(\alpha)$ .

Now, to test the accuracy of this method, many more repetitions of this testing and limit estimating are done. This yields a number of lower confidence limits that can be compared to the true system reliability. As stated before, one would expect  $100(1-\alpha)\%$  of these  $100(1-\alpha)\%$  lower confidence limits to include the true system reliability.

The outline of this procedure is as follows:

- (1) Establish the value of  $p_i$  for each component and the number of tests to subject each component to,  $NT_i$ ,
- (2) For each component in turn, draw  $NT_i$  random variables from Uniform [0,1],
- (3) For each component, compare these  $NT_i$  random variables to  $p_i$ ; for each random variable that is greater than  $p_i$ , record a failure,
- (4) Calculate the first estimate

$$\hat{p}_i = 1 - \frac{F_i}{NT_i} \text{ or } 1 - \frac{F'_i}{NT_i} ,$$

$$\hat{q}_i = 1 - \hat{p}_i ,$$

$$\text{asymptotic variance} = \frac{\hat{p}_i \hat{q}_i}{N T_i} ,$$

(5) Draw a random variable from  $N(0,1)$  for each component,

(6) Obtain a second estimate,  $\hat{\bar{p}}_i$ , by taking the  $N(0,1)$  random variable, multiply by the asymptotic standard deviation, and add it to  $\hat{p}_i$  for each component,

(7) With the components in a logically complex system, calculate a system reliability estimate,  $\hat{R}_s = \prod_{i=1}^k \hat{\bar{p}}_i$ ,

(8) Repeat steps (5) through (7) 999 times,

(9) Rank order these 999 estimates of  $R_s$ ,

(10) Extract the  $100(1-\alpha)$  percentile value to obtain a  $100(1-\alpha)\%$  lower confidence limit,

(11) Repeat steps (2) through (10) 1000 times to obtain 1000 of these  $100(1-\alpha)\%$  lower confidence limits,

(12) Determine how many of these lower confidence limits are covering the true system reliability,  $R_s = \prod_{i=1}^k p_i$ .

The above procedure was applied to systems containing 2 and 3 components in series. The computer program used (see Appendix C) shows that, with minor adjustments, larger systems can be examined. In these 2 and 3 component systems, known probabilities of success,  $p_i$ , of 0.6, 0.8, 0.9, and 0.95 were examined to find 80%, 90%, and 95% lower confidence limits. Table V and Table VI show the number of lower confidence limits that contained the true system reliability for each of the systems with various values of  $p_i$ ,  $\alpha$ , and number of tests.

TABLE V  
Two Component System

		% of Limits Covering R <sub>S</sub>			No. of Tests			% of Limits Covering R <sub>S</sub>		
		P <sub>i</sub> = .6		P <sub>i</sub> = .8	P <sub>i</sub> = .6		P <sub>i</sub> = .8	P <sub>i</sub> = .6		P <sub>i</sub> = .8
		α = .2	α = .2	α = .2	α = .1	α = .1	α = .1	α = .05	α = .05	α = .05
α = .1	20	80	80	80	90	91	91	95	95	95
	50	80	80	80	90	91	91	95	95	95
	100	80	80	80	90	91	91	95	95	95
α = .05	20	90.3	90.3	90.3	91	91	91	95	95	95
	50	90.3	90.3	90.3	91	91	91	95	95	95
	100	90.3	90.3	90.3	91	91	91	95	95	95
P <sub>i</sub> = .9	20	95.1	95.1	95.1	95	95	95	99	99	99
	50	95.1	95.1	95.1	95	95	95	99	99	99
	100	95.1	95.1	95.1	95	95	95	99	99	99
α = .2	20	83	83	83	80	80	80	85	85	85
	50	83	83	83	81	81	81	85	85	85
	100	83	83	83	81	81	81	85	85	85
α = .1	20	88	88	88	90	90	90	95	95	95
	50	88	88	88	90	90	90	95	95	95
	100	88	88	88	90	90	90	95	95	95
α = .05	20	95	95	95	92.6	92.6	92.6	98	98	98
	50	95	95	95	93.8	93.8	93.8	98	98	98
	100	95	95	95	93.8	93.8	93.8	98	98	98

TABLE VI  
Three Component System

No. of Tests			% of Limits Covering $R_S$			No. of Tests			% of Limits Covering $R_S$		
$P_i = .6$			$P_i = .8$			$P_i = .8$			$P_i = .9$		
$\alpha = .2$			$\alpha = .2$			$\alpha = .1$			$\alpha = .05$		
20	50	100	90	81	85	20	50	100	20	50	100
$\alpha = .1$	20	50	94	90	94	20	50	100	92	91	90.4
	50	100									
$\alpha = .05$	20	50	97	95.4	96	20	50	100	94.1	97	95.5
	50	100									
$P_i = .9$	20	50	80	80	82	20	50	100	83.2	77.5	78.7
$\alpha = .2$	50	100									
$\alpha = .1$	20	50	90.3	89	94	20	50	100	94.2	88.7	89.1
	50	100									
$\alpha = .05$	20	50	93	97	94.3	20	50	100	95.8	90.5	93.5
	50	100									

Close examination of Table V and Table VI points out that accuracy declines as  $p_i$  gets larger and as  $\alpha$  gets smaller. These results verify, as Mann and Grubbs mentioned (Ref 18:343), that an upward biasness might exist.

By studying the computer program in Appendix C, the reader should note several points. At higher values of  $p_i$ , some tests will exhibit no failures. Hence, it is possible that  $\hat{p}_i$  can be greater than unity. To eliminate this event,  $\hat{p}_i$  is chosen to be the minimum  $[N(\hat{p}_i, \frac{\hat{p}_i q_i}{NT_i}, 1)]$ . Also, the reader should note that, on output, he is provided with an average lower confidence limit. In the generation of the 1000 lower confidence limits, they are averaged to produce the most accurate estimate.

To illustrate the utility and simplicity of this technique, consider the mission profile of an air-to-ground missile. Assume that it has eight independent mission stages with associated probabilities of success:

- (1) Release from aircraft,  $p_1 = 0.95$
- (2) Air start,  $p_2 = 0.95$
- (3) Deploy flight control surfaces,  $p_3 = 0.9$
- (4) Arrive at battle area,  $p_4 = 0.95$
- (5) Terrain following/avoidance,  $p_5 = 0.85$
- (6) Penetrate defenses,  $p_6 = 0.75$
- (7) Hit target,  $p_7 = 0.95$
- (8) Detonate,  $p_8 = 0.95$

These stages are analogous to a system with eight components in series. By multiplying all of the  $p_i$ 's, the true

system reliability (probability of killing the target) is determined to be 0.42059. Exercising the proposed method with 100 tests per component with the above  $p_i$ 's, a 90% lower confidence limit for system reliability is 0.3665. In this particular case, 91.3% of the 1000 limits cover the true reliability. Therefore, we would say that it is a very accurate estimate.

Another example of the usefulness and simplicity of this technique is shown by estimating the lower confidence limits for the two simple parallel-series systems shown in Figures 1 and 2. In analyzing the three component system shown in Figure 1, the probability of success for each component,  $p_i$ , was chosen to be 0.9; and each component was tested 100 times. This design yields a system reliability of 0.891. The estimating technique provides an 80% lower confidence limit of 0.86842 with the lower limits covering the system reliability in 75% of the 1000 Monte Carlo repetitions. This would indicate that the LCL estimate is slightly high.

The five component system in Figure 2 was assigned the following  $p_i$ 's:

- (1)  $p_1 = 0.95$
- (2)  $p_2 = 0.75$
- (3)  $p_3 = p_4 = 0.90$
- (4)  $p_5 = 0.80$

Calculations yield a system reliability of 0.9006 and a 90% lower confidence limit of 0.8671. These limits cover the

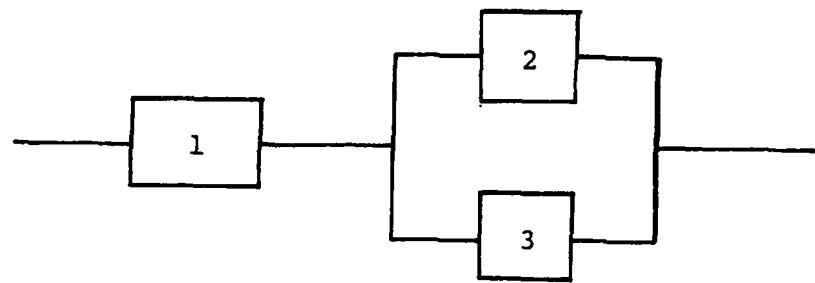


Figure 1. Three Component System

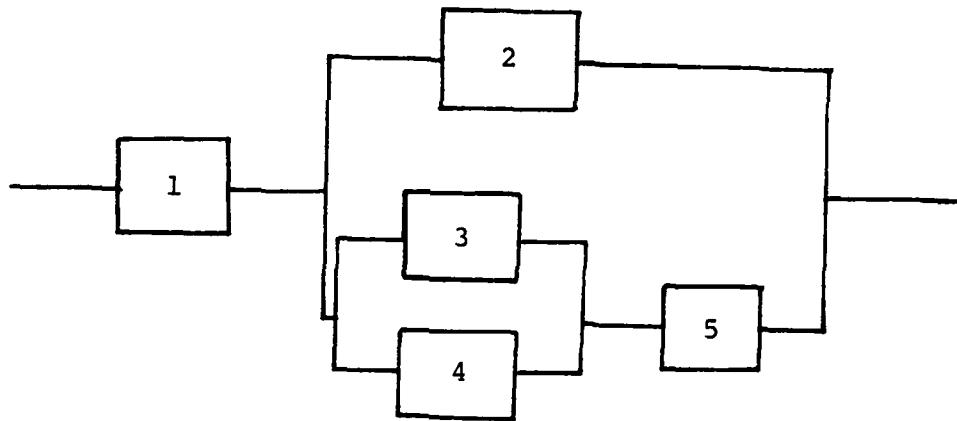


Figure 2. Five Component System

system reliability in 87.8% of the 1000 cases which indicates again that the LCL estimate is slightly optimistic.

## V. Conclusions and Recommendations

Stated very simply, it has been shown that this method of estimating lower confidence limits is accurate, practical, and simple to use. One need only supply the component reliabilities, the number of component tests, and the desired level of confidence; and he obtains, not only an estimated lower confidence limit of the system reliability, but also an indication of how accurate this estimate is.

It is worth re-emphasizing that, where most of the other techniques are not valid in the case of zero-failures, this method accurately and easily accommodates such a situation. This method is also not restricted to series systems; it can easily handle parallel configurations. A logical next step would be to incorporate such a method into a system where other components exhibit different failure distributions as in the work done by Moore, Harter, and Snead (Ref 20).

This next generation of systems to be analyzed obviously provokes one to imagine all the possible applications. These methods could be applied to an entire spectrum of systems from simple hardware items to strategic mission planning.

## Bibliography

1. Bernhoff, O. A. "Confidence Limits for System Reliability Based on Component Test Data," M.S. thesis (GRE/MA/63-1), Air Force Institute of Technology, Wright-Patterson Air Force Base, Ohio, 1963, AD425845.
2. Berkbigler, K. P. and Byers, J. K. "System Reliability: Exact Bayesian Intervals Compared with Fiducial Intervals," IEEE Transactions on Reliability, Vol. R-24, August 1975, pp 199-204.
3. Buehler, R. J. "Confidence Intervals for the Product of Two Binomial Parameters," Journal of the American Statistical Association, Vol. 52, December 1952, pp 482-493.
4. Burnett, T. L. and Wales, B. A. "System Reliability Confidence Limits," Proceedings of the Seventh National Symposium on Reliability and Equality Control, 1961, pp 118-128.
5. Easterling, R. G. "Approximate Confidence Limits for System Reliability," Journal of the American Statistical Association, Vol. 67, pp 220-222.
6. El Mawaziny, A. H. and Buehler, R. J. "Confidence Limits for the Reliability of a Series System," Journal of the American Statistical Association, Vol. 62, pp 1452-1459.
7. Gatliffe, T. R. "Accuracy Analysis for a Lower Confidence Limit Procedure for System Reliability," M.S. thesis, Naval Postgraduate School, Monterey, California, 1976.
8. Johnson, J. R. "Confidence Interval Estimation of the Reliability of Multicomponent Systems Using Component Test Data," US Army Aberdeen Research and Development Center, Technical Report No. 3, AD699809, 1969.
9. Lannon, R. G. "A Monte Carlo Technique for Approximating System Reliability Confidence Limits Using the Weibull Distribution," M.S. thesis (GAM/MA/72-2), Wright-Patterson Air Force Base, Ohio, 1972, AD743633.
10. Levy, L. L. and Moore, A. H. "A Monte Carlo Technique for Obtaining System Reliability Confidence Limits from Component Test Data," IEEE Transactions on Reliability, Vol. R-16, September 1967, pp 69-72.

11. Lipow, M. and Riley, J. Tables of Upper Confidence Bounds on Failure Probability of 1, 2, and 3 Component Serial Systems, Vols. I and II, Space Technology Laboratories, AD609100 and AD636718.
12. Lloyd, D. K. and Lipow, M. Reliability: Management, Methods, and Mathematics, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1962.
13. Locks, M. O. "Monte Carlo Bayesian System Reliability--and MTBF--Confidence Assessment," Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio, AFFDL-TR-75-144.
14. Locks, M. O. "Monte Carlo Bayesian System Reliability--and MTBF--Confidence Assessment," Vol. I: Theory, Vol. II: SPARCS-2 Users Manual, Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio, AFFDL-TR-78-18.
15. Lutton, S. C. "A Monte Carlo Technique for Approximating System Reliability Confidence Limits from Component Failure Test Data," M.S. thesis (GRE/MA/67-9), Air Force Institute of Technology, Wright-Patterson Air Force Base, Ohio, 1967.
16. Mann, N. R. "Computer-Aided Selection of Prior Distributions for Generating Monte Carlo Confidence Bounds on System Reliability," Naval Research Logistics Quarterly, Vol. 17, pp 41-54.
17. Mann, N. R. Methods for Statistical Analysis of Reliability and Life Data, 1974, pp 487-528.
18. Mann, N. R. and Grubbs, F. E. "Approximately Optimum Confidence Bounds for System Reliability Based on Component Test Data," Technometrics, Vol. 16, No. 3, August 1974, pp 335-347.
19. Moore, A. H. "Extension of Monte Carlo Technique for Obtaining System Reliability Confidence Limits from Component Test Data," Proceedings of National Aerospace Electronics Conference, May 1965, pp 459-463.
20. Moore, A. H., Harter, H. L. and Snead, R. C. "A Comparison of Monte Carlo Techniques for Obtaining System Reliability Confidence Limits from Component Test Data," (submitted for publication).
21. Orkand, D. S. A Monte Carlo Method for Determining Lower Confidence Limits for System Reliability on the Basis of Sample Component Data, ORDBB-VC-4, June 1960.

22. Springer, M. D. and Thompson, W. E. "Bayesian Confidence Limits for the Product of N Binomial Parameters," Biometrika, Vol. 53, December 1966, pp 611-613.
23. Springer, M. D. and Thompson, W. E. "Bayesian Confidence Limits for Reliability of Redundant Systems When Tests Terminated at First Failure," Technometrics, Vol. 10, February 1968, pp 29-36.
24. Steck, G. P. Upper Confidence Limits for the Failure Probability of Complex Networks, SC-433(TR), Sandia Corp.
25. Woods, M. and Borsting, J. "A Method for Computing Lower Confidence Limits on System Reliability Using Component Failure Data with Unequal Sample Sizes," US Naval Postgraduate Report, NPS55 wo/Bg 8061A.

## Appendix A

### Case Where $\hat{s}=0$

In Gatliffe's method (Ref 7), he must deal with the case of  $\hat{s}=0$ . In his study, the number of mission successes,  $W$ , can be related to mission reliability,  $R_s$ , by

$$\sum_{j=W}^N \binom{N}{j} R_s^j (1-R_s)^{N-j} = \text{Prob } (\# \text{ successes} \geq W) ,$$

where  $N$  = number of mission trials. The 100(1- $\alpha$ )% Lower Confidence Limit estimate for system reliability would be the solution for  $R^*$  in

$$\sum_{j=W}^N \binom{N}{j} (R^*)^j (1-R^*)^{N-j} = \alpha$$

which, when  $W=N$ , reduces to

$$(R^*)^N = \alpha \text{ or } R^* = (\alpha)^{1/N} .$$

$$\text{Now, } \hat{R}^*(\alpha) = \exp \left[ \frac{-2\hat{r}\hat{s}}{2X(2\hat{r}, 1-\alpha)} \right] = (\alpha)^{1/N}$$

$$\text{where } \hat{s} = \sum T_i, \hat{r} = \max \left[ 1, \frac{\left( \sum T_i \right)^2}{\sum \left( \frac{T_i}{N_i} \right)} \right]$$

which reduces to  $\hat{s} = \hat{T}_i$  and  $\hat{r} = \max(1, \hat{T}_i N_i)$ , since only one component contributes non-zero failures.

Now if  $\hat{r} = 1.0$ , then  $\hat{T}_i N_i \leq 1$  implies that

$$\hat{T}_i \leq \frac{1}{N_i}$$

$$\hat{T}_i = A_i \hat{Q}_i + B_{i/2} \hat{Q}_i^2$$

$$A_i = \frac{2N_i - 3}{2N_i - 2}, \quad B_{i/2} = \frac{N_i}{N_i - 1}, \quad \hat{Q}_i = \frac{F_i}{N_i}.$$

$$\text{Thus, } \left(\frac{B_i}{2N_i^2}\right) F_i^2 + \left(\frac{A_i}{N_i}\right) F_i - \hat{T}_i = 0 \quad \text{or}$$

$$\left(\frac{N_i}{2(N_i - 1)(N_i^2)}\right) F_i^2 + \left(\frac{2N_i - 3}{2(N_i - 1)(N_i)}\right) F_i - \frac{1}{N_i} \leq 0$$

$$F_i^2 + (2N_i - 3)F_i - 2(N_i - 1) \leq 0$$

$$(F_i - 1)(F_i + 2(N_i - 1)) \leq 0.$$

The only positive values for  $F_i$  that can satisfy this equation are between zero (0) and one (1). Also, for any size  $N_i$ ,  $2\hat{r} = 2$ . Hence,

$$\exp\left[\frac{-2\hat{r}\hat{s}}{\chi^2(2\hat{r}, 1-\alpha)}\right] = (\alpha)^{1/N}$$

$$\hat{s} = \frac{\chi^2(2\hat{r}, 1-\alpha)}{2N} (\ln \alpha) = \frac{A_i F_i^*}{N} + \frac{B_i (F_i^*)^2}{2N^2}$$

which implies  $(F_i^*)^2 + (2N-3)F_i^* + (N-1) \ln \alpha (\chi^2(2\hat{r}, 1-\alpha)) = 0$ .

Solving for  $F_i^*$  as a function of  $N$  and  $\alpha$ , Gatliffe obtains

<u>N</u>	<u><math>\alpha = .2</math></u>	<u><math>\alpha = .1</math></u>	<u><math>\alpha = .05</math></u>
5	.3888	.2670	.1713
10	.3721	.2531	.1610
20	.3652	.2475	.1570
100	.3603	.2435	.1542
1000	.3593	.2427	.1536

## Appendix B

### Computer Program to Determine Limits

The computer program contained herein determines the  $100(1-\alpha)\%$  lower confidence limits on system reliability. It is written in FORTRAN IV and utilizes AFIT and IMSL subroutines. At the beginning, each variable is defined. Throughout the program, major sections are introduced with comments that explain what is being done so as to enhance the flow of the program.

Input: The first data card is in free field form containing:

- (1) the number of components in the system,
- (2) the confidence level ( $\alpha = 0.05$  means a 95% level),
- (3) DSEED which is the double precision random number seed used in the subroutines,
- (4) the number of simulations to be performed (999).

Following this card is a card for each component, in format, that establishes the number of tests and number of failures for that component.

Output: On output is displayed:

- (1) the number of components,
- (2) the number of tests and failures for each component,
- (3) the confidence level,
- (4) the  $100(1-\alpha)\%$  lower confidence limit.

```

      P=SYSTEM RELIABILITY (INPUT=/.00,OUTPUT)
      NITEST=NITEST(1),FSYSU(1,0),FITEST(1),NITEST(1,0),
      NC0M(1,0),SIGIAT(1,0),T(1,0)
      T=TIME P EQUALS 1.0E-07

      NITEST= # OF COMPONENT I TESTED
      FITEST= # OF COMPONENT I FAILED IN TEST
      PIHAT= FIRST ESTIMATE OF P FOR COMPONENT I
      PIHAT2= SECOND ESTIMATE OF P FOR COMPONENT I
      FSYSU= RELIABILITY OF SYSTEM ON MONTE CARLO SIM
      NC0M= # OF COMPONENTS IN THE SYSTEM
      ALPHA= 100(1-ALPHA)% CONFIDENCE LIMIT
      FI= EQUIVALENT FAILURES
      OIHAT= FIRST ESTIMATE OF Q FOR COMPONENT I:O=1-PI
      SIGIAT= ESTIMATE OF VARIANCE
      DSEED= SEED FOR NORMAL R.V. GENERATION
      R= OUTPUT VECTOR OF NORMAL R.V.'S
      NSIM= # OF MONTE CARLO SIMULATIONS
      KALPHA= THAT VALUE CORRESPONDING TO LIMIT

      FORMAT (1X,I4,1X,FF.3)
      FORMAT (F8.4)
      READ 1, (NC0M,ALPHA,DSEED,NSIM)
      IF (EOF(SLINPUT).NE.0.) STOP "END OF PROGRAM"

      NO. OF COMPONENTS TESTED AND NO. OF FAILURES
      DO 10 I=1,NC0M
      READ 2, (NITEST(I),FITEST(I))
      CONTINUE
      PRINT *, "INPUT CHECK...NO. OF COMP= ",NC0M," NO. OF TESTS AND FAIL"
      PRINT *, ""
      PRINT 2,(NITEST(I),FITEST(I),I=1,NC0M)

      ASSIGNING EQUIVALENT FAILURES WITH GATLIFFE'S METHOD
      IF ((ALPHA.EQ.1.0)) FI=.780
      IF ((ALPHA.EQ.0.9)) FI=.245
      IF ((ALPHA.EQ.0.1)) FI=.155
      DO 20 I=1,NC0M
      IF (FITEST(I).EQ.0.) FITEST(I)=FI
      CONTINUE

      CALCULATING P, Q, AND VARIANCE
      DO 30 I=1,NC0M
      PIHAT(I)=1-(FITEST(I))/(NITEST(I))
      OIHAT(I)=1-PIHAT(I)
      SIGIAT(I)=PIHAT(I)*OIHAT(I)/NITEST(I)
      CONTINUE

      GENERATING NORMAL RANDOM VARIABLES AND SECOND ESTIMATE OF P
      DO 50 J=1,NSIM
      CALL GGNML(DSEED,NC0M,F)
      DO 40 I=1,NC0M
      PIHAT2(I)=R(I)+SQRT(SIGIAT(I))+PIHAT(I)
      CONTINUE

      CALCULATING SYSTEM RELIABILITY (SERIES)
      DO 45 I=2,NC0M

```

```
      R1=1.0(I)=R1HAT2(I)*RTHAT2(I-1)
      CONTINUE
      RSYSU(J)=R1HAT2(NCOM)
      CONTINUE

      PRINT*, 'ARRANGING SYSTEM RELIABILITY IN ASCENDING ORDER'
      CALL SCRT(NGIT1,RSYR1)
      IF (ALPHA.EQ.0.1) KALPHA=100
      IF (ALPHA.EQ. 1.0) KALPHA=50
      PRINT*, " "
      PRINT*, "NO. OF COMPONENTS = ",NCOM
      DO TO 1 I=1,NCOM
      PRINT*, "NO. OF COMPONENT ",I," TESTED=",NITEST(I)
      PRINT*, "NO. OF FAILURES FOR THIS COMPONENT WAS ",FTEST(I)
      CONTINUE
      T=1.0-ALPHA
      PRINT*, "LOWER LIMIT OF ",T," IS "
      PRINT 3, (RSYSU(KALPHA))
      GO TO 1
      END
```

## Appendix C

### Computer Program to Verify Accuracy

The computer program contained herein determines the  $100(1-\alpha)\%$  lower confidence limit on system reliability and determines how accurate this limit is. It is written in FORTRAN IV and utilizes AFIT and IMSL subroutines. At the beginning, each variable is defined. Throughout the program, major sections are introduced with comments that explain what is being done so as to enhance the flow of the program.

Input: The first data card is in free field form containing:

- (1) the number of components in the system,
- (2) the confidence level ( $\alpha = 0.05$  means a 95% level),
- (3) DSEED which is the double precision random number seed used in the subroutines,
- (4) the number of simulations to be performed (999),
- (5) the number of lower limits to be generated (1000).

Following this card is a card for each component, in format, that establishes the true probability of success and number of tests for that component.

Output: On output is displayed:

- (1) the true probability of success and number of tests for each component,
- (2) the true system reliability,
- (3) the confidence level and number of limits,

- (4) the averaged  $100(1-\alpha)\%$  lower confidence limit,
- (5) the accuracy of the limit (the percent of the total limits that covered the true system reliability).

```

PROGRAM BINREL1 (INPUT,OUTPUT)
DIMENSION PIHAT(10),RSYSJ(1000),R(1000),FITEST(11),NITEST(10),
1CIHAT(10),SIGIHAT(11),PIHAT2(3,101),TRUEP(10),RTEST(1000)
DOUBLE PRECISION DSEED

C ****
C NITEST= # OF COMPONENT I TESTED
C FITEST= # OF COMPONENT I FAILED IN TEST
C PIHAT= FIRST ESTIMATE OF P FOR COMPONENT I
C PIHAT2= SECOND ESTIMATE OF P FOR COMPONENT I
C RSYSJ= RELIABILITY OF SYSTEM ON MONTE CARLO SIM J
C NCNM= # OF COMPONENTS IN THE SYSTEM
C ALPHA= 1e0(1-ALPHA)% CONFIDENCE LIMIT
C FI= EQUIVALENT FAILURES
C OIHAT= FIRST ESTIMATE OF Q FOR COMPONENT I:Q=1-P
C SIGIHAT= ESTIMATE OF VARIANCE
C DSEED= SEED FOR NORMAL R.V. GENERATION
C R= OUTPUT VECTOR OF NORMAL R.V.S
C NSIM= # OF MONTE CARLO SIMULATIONS
C KALPHA= THAT VALUE CORRESPONDING TO LIMIT
C NOVER=NO OF LCL OVER THE TRUE RELIABILITY
C NLIM=NUMBER OF LOWER CONFIDENCE LIMITS
C TRSYS=THE TRUE SYSTEM RELIABILITY
C RTEST=ARRAY CONTAINING TEST RESULTS OF THE PI'S
C REL= RELIABILITY OF THE SYSTEM FOR THAT SIMULATION
C CUMLM= ADDS ALL THE LCL'S
C AVGLM= AVERAGES THESE LIMITS OVER THE TOTAL NO. GENERATED
C ****
C
2 FORMAT (2X,F6.4,1X,I3)
3 FORMAT (F8.6)
5 READ*, NCNM,ALPHA,DSEED,NSIM,NLIM
IF (EOF(5) .NE. 0.0) STOP "END OF PROGRAM"
C ****
C TRUE PROB OF SUCCESS AND NO. OF TESTS FOR EACH COMPONENT
C
      TRSYS=1.0
      DO 10 I=1,NCNM
      READ 2, (TRUEP(I),NITEST(I))
      TRSYS=TRSYS*TRUEP(I)
      PRINT*, "THE TRUE VALUE OF P FOR COMPONENT ",I," = ",TRUEP(I)
      PRINT*, "NUMBER OF TESTS ON COMPONENT ",I," = ",NITEST(I)
10    CONTINUE
      PRINT*, " "
      PRINT*, "THE TRUE SYSTEM RELIABILITY IS ",TRSYS
C ****
C NOW TESTING COMPONENTS WITH UNIFORM FUNCTION
C
      NOVEP=0
      CUMLM=0.0
      DO 100 LIMITS=1,NLIM
      DO 20 M=1,NCNM
      N=NITEST(M)
      CALL GGUBS(DSEED,N,RTEST)
C

```

```

C*****+
C   FTEST CONTAINS TEST VALUES FOR COMPONENTS
C
      FITEST(M)=0.0
      DO 30 K=1,N
          IF(FTEST(K).GT.TRUEP(M)) FITEST(M)=FITEST(M)+1
30      CONTINUE
C
C*****+
C   ASSIGNING EQUIVALENT FAILURES WITH GATLIFFE'S METHOD
C
      IF (ALPHA.EQ.0.2) FT=.360
      IF (ALPHA.EQ.0.1) FT=.245
      IF (ALPHA.EQ.0.05) FI=.155
      IF (FITEST(M).EQ.0.0) FITEST(M)=FI
C
C*****+
C   CALCULATING ESTIMATES OF P, Q, AND VARIANCE
C
      PIHAT(M)=1.0-(FITEST(M))/(NITEST(M))
      QIHAT(M)=1.0-PIHAT(M)
      SIGIHAT(M)=PIHAT(M)*QIHAT(M)/NITEST(M)
20      CONTINUE
C
C*****+
C   GENERATING NORMAL RANDOM VARIABLES AND SECOND ESTIMATE OF P
C   CALCULATING SYSTEM RELIABILITY (SERIES)
C
      DO 50 I=1,NCOM
      CALL GGNML(DSEED,NSIM,F)
      DO 40 J=1,NSIM
          PIHAT2(I,J)=PIHAT(I)-(F(J)*SQRT(SIGIHAT(I)))
          IF(PIHAT2(I,J).GT.1.0) PIHAT2(I,J)=1.0
40      CONTINUE
50      CONTINUE
      DO 60 J=1,NSIM
          REL=1.0
          DO 70 I=1,NCOM
              REL=REL*PIHAT2(I,J)
70      CONTINUE
          RSYSJ(J)=REL
60      CONTINUE
C
C*****+
C   ARRANGING SYSTEM RELIABILITY IN ASCENDING ORDER
C
      CALL SCRT(NSIM,RSYSJ)
      KALPHA=ALPHA*NSIM
      T=1.0-ALPHA
      IF(RSYSJ(KALPHA).GT.TRSYS) NOVER=NOVER+1
      CUMLM=CUMLM+RSYSJ(KALPHA)
100     CONTINUE
      AVGLM=CUMLM/NLIM
      PERCENT=NLIM-NOVER
      PERCNT=(PERCENT/NLIM)*100.0
      PRINT*, "NUMBER OF LOWER LIMITS AT ALPHA= ",T," IS ",NLIM
      PRINT*, "AVERAGE LOWER LIMIT OF SYSTEM RELIABILITY IS ",AVGLIM

```

42

```
PRINT*, "PERCENT OF LOWER LIMITS COVERING THE TRUE REL IS ", PERCNT
PRINT*, " "
PRINT*, " "
GO TO 5
END
```

VITA

Roy Eugene Rice was born on 24 January 1953 in Little Rock, Arkansas. He graduated from high school in Lonoke, Arkansas, in 1971 and attended the United States Air Force Academy. Upon graduation in June 1975, he received a Bachelor of Science degree in Mathematics and a commission in the U. S. Air Force. He was then placed on active duty at Tinker Air Force Base, Oklahoma, where he was a Reliability Engineer. Some of the systems on which he worked were the B-1 Bomber, B-52, KC-135, E-3A (AWACS), E-4, and the Advanced Aerial Refueling Boom. After approximately three years at Tinker, he entered the School of Engineering, Air Force Institute of Technology in June 1978.

Permanent address: 318 Barker Street  
Lonoke, Arkansas 72086

## UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFIT/GOR/MA/79D-8	2. GOVT ACCESSION NO. <i>AD-A113 997</i>	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) INCORPORATION OF ASYMPTOTIC NORMALITY PROPERTIES OF THE BINOMIAL DISTRIBUTION INTO A MONTE CARLO TECHNIQUE FOR ESTIMATING LOWER CONFIDENCE LIMITS ON SYSTEM RELIABILITY	5. TYPE OF REPORT & PERIOD COVERED M.S. Thesis	
7. AUTHOR(s) Roy E. Rice Capt USAF	6. PERFORMING ORG. REPORT NUMBER	
9. PERFORMING ORGANIZATION NAME AND ADDRESS Air Force Institute of Technology (AFIT-EN) Wright-Patterson AFB, Ohio 45433	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS	
11. CONTROLLING OFFICE NAME AND ADDRESS Electronic Warfare Division Aeronautical Systems Division Wright-Patterson AFB, Ohio 45433	12. REPORT DATE December 1979	13. NUMBER OF PAGES 52
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report) Unclassified	
16. DISTRIBUTION STATEMENT (of this Report)  Approved for public release; distribution unlimited		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES Approved for public release; IAW AFR 190-17 <i>578</i>		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number)  Confidence limit                    Monte Carlo simulation Binomial distribution Reliability Asymptotic normality		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Several methods for estimating lower confidence limits were examined. A proposed technique was developed and used to obtain limits for selected systems. These limits were compared to those obtained by other methods. Then the proposed method was tested to verify its accuracy. Results indicate that the proposed method is simple to understand, easy to implement, and accurate.		

